## EE 330 Lecture 20

Bipolar Device Modeling

## Fall 2023 Exam Schedule

Exam 1 Friday Sept 22
Exam 2 Friday Oct 20
Exam 3 Friday Nov. 17
Final Monday Dec 11 12:00-2:00 p.m.

## Review from Last Lecture

## Bipolar Transistors



With proper doping and device sizing these form Bipolar Transistors

Review from Last Lecture

## Bipolar Operation



Some will recombine with holes and contribute to base current and some will be attracted across BC junction and contribute to collector

Size and thickness of base region and relative doping levels will play key role in percent of minority carriers injected into base contributing to collector current

# Bipolar Operation 

Consider npn transistor - Forward Active Operation tentatively: $\mathrm{V}_{\mathrm{BE}}>0.4 \quad \mathrm{~V}_{\mathrm{BC}}<0.4$


$$
\left.\begin{array}{l}
\mathrm{I}_{\mathrm{C}}+\mathrm{I}_{\mathrm{B}}=-\mathrm{I}_{\mathrm{E}} \\
\mathrm{I}_{\mathrm{C}}=-\alpha \mathrm{I}_{\mathrm{E}}
\end{array}\right\} \quad \begin{aligned}
& \mathrm{I}_{\mathrm{C}}=\frac{\alpha}{1-\alpha} \mathrm{I}_{\mathrm{B}} \\
& \beta=\frac{\alpha}{1-\alpha}
\end{aligned}
$$

$\beta$ is typically very large

$$
\mathrm{I}_{\mathrm{C}}=\beta \mathrm{I}_{\mathrm{B}}
$$

often $50<\beta<999$

# Bipolar Operation 

Consider npn transistor - Forward Active Operation tentatively: $\mathrm{V}_{\mathrm{BE}}>0.4 \quad \mathrm{~V}_{\mathrm{BC}}<0.4$

$\beta$ is typically very large
Bipolar transistor can be thought of as current amplifier with a large current gain In contrast, MOS transistor is inherently a tramsconductance amplifier
Current flow in base is governed by the diode equation

$$
\begin{aligned}
& \mathrm{I}_{\mathrm{B}}=\widetilde{I}_{S} \mathrm{e}^{\frac{\mathrm{V}_{\mathrm{BE}}}{\mathrm{~V}_{\mathrm{t}}}} \\
& \mathrm{I}_{\mathrm{C}}=\beta \widetilde{I}_{S} \mathrm{e}^{\frac{\mathrm{V}_{\mathrm{BE}}}{\mathrm{~V}_{\mathrm{t}}}}
\end{aligned}
$$

Collector current thus varies exponentially with $V_{B E}$

## Review from Last Lecture

## Preliminary Comparison of MOSFET and BJT

(Saturation vs Forward Active)

$I_{D}$ independent of $V_{D S}$

$I_{C}$ independent of $V_{C E}$

- The BJT I/O relationship is exponential in contrast to square-law for MOSFET
- Provides a very large "gain" for the BJT (assuming input is voltage and output is current)
- This property is very useful for many applications


## Bipolar Models

## Simple dc Model



Following convention, pick $I_{C}$ and $I_{B}$ as dependent variables and $V_{B E}$ and $V_{C E}$ as independent variables

## Sinnele OCNOA

## Consider npn transistor - Forward Active Operation

Summary:

$$
\left.\begin{array}{l}
\mathrm{I}_{\mathrm{B}}=\widetilde{I}_{S} \mathrm{e}^{\frac{\mathrm{V}_{\mathrm{BE}}}{\mathrm{~V}_{\mathrm{t}}}} \\
\mathrm{I}_{\mathrm{C}}=\beta \tilde{I}_{S} \mathrm{e}^{\frac{\mathrm{V}_{\mathrm{BE}}}{\mathrm{~V}_{\mathrm{t}}}} \\
\mathrm{~V}_{\mathrm{t}}=\frac{\mathrm{kT}}{\mathrm{q}}
\end{array}\right\}
$$

$$
\xrightarrow[\substack{+\mathrm{V}_{\mathrm{BE}}}]{\substack{\mathrm{I}_{\mathrm{B}}}} \int_{\mathrm{V}}^{\mathrm{I}}
$$

This has the properties we are looking for but the variables we used in introducing these relationships are not standard

It can be shown that $\widetilde{I}_{S}$ is proportional to the emitter area $\mathrm{A}_{\mathrm{E}}$
Define $J_{S}$ by $\widetilde{I}_{S}=\beta^{-1} \mathbf{J}_{\mathbf{S}} \mathbf{A}_{\mathbf{E}}$ and substitute this into the above equations

## Simple dc model

$$
\begin{aligned}
& \text { npn transistor - Forward Active Operation } \\
& \mathrm{I}_{\mathrm{B}}=\widetilde{I}_{S} \mathrm{e}^{\frac{\mathrm{V}_{\mathrm{BE}}}{\mathrm{~V}_{\mathrm{t}}}} \\
& \left.\mathrm{I}_{\mathrm{C}}=\beta \widetilde{I}_{S} \mathrm{e}^{\frac{\mathrm{V}_{\mathrm{BE}}}{V_{\mathrm{t}}}}\right\} \\
& V_{t}=\frac{k T}{q} \quad \mathrm{k} / \mathrm{q}=8.62 \times 10^{-5} \\
& I_{B}=\frac{J_{S} A_{E}}{\beta} e^{\frac{V_{B E}}{V_{t}}} \\
& I_{C}=J_{S} A_{E} e^{\frac{V_{B E}}{V_{t}}} \\
& V_{t}=\frac{k T}{q} \\
& \text { Standard Notation : } \\
& \beta \text { moved to } \mathrm{I}_{\mathrm{C}} \text { equation }
\end{aligned}
$$

$J_{S}$ is termed the saturation current density
Process Parameters: $J_{S}, \beta$
Design Parameters: $A_{E}$
Environmental parameters and physical constants: k,T,q
At room temperature, $V_{t}$ is around 26 mV
$J_{S}$ very small - around $.25 f A / u^{2}$ at room temperature

## Simple dc model

npn transistor - Forward Active Operation

$$
\left.\begin{array}{l}
I_{B}=\frac{J_{S} A_{E}}{\beta} e^{\frac{V_{B E}}{V_{t}}} \\
I_{C}=J_{S} A_{E} e^{\frac{V_{B E}}{V_{t}}} \\
V_{t}=\frac{k T}{q}
\end{array}\right\}
$$

As with the diode, the parameter $J_{S}$ is highly temperature dependent

$$
J_{s}=J_{s x}\left[T^{m} e^{\frac{-v_{0}}{v_{i}}}\right]
$$

Typical values for parameters: $\quad J_{\mathrm{SX}}=20 \mathrm{~mA} / \mu^{2}, \mathrm{~V}_{\mathrm{G} 0}=1.17 \mathrm{~V}, \mathrm{~m}=2.3$
The parameter $\beta$ is also somewhat temperature dependent but much weaker temperature dependence than $\mathrm{J}_{\mathrm{Sx}}$.

## Transfer Characteristics

npn transistor - Forward Active Operation

$$
\begin{aligned}
& J_{S}=.25 \mathrm{fA} / \mathrm{u}^{2} \\
& A_{E}=400 \mathrm{u}^{2}
\end{aligned}
$$


$V_{B E}$ close to 0.6 V for a two decade change in $\mathrm{I}_{\mathrm{C}}$ around 1 mA

## Transfer Characteristics

npn transistor - Forward Active Operation

$$
\begin{aligned}
& J_{S}=.25 \mathrm{fA} / \mathrm{u}^{2} \\
& A_{E}=400 \mathrm{u}^{2}
\end{aligned}
$$


$V_{B E}$ close to 0.6 V for a four decade change in $\mathrm{I}_{\mathrm{C}}$ around 1 mA

# Simple dc model 

npn transistor - Forward Active Operation
Output Characteristics


## Simple dc model

Better Model of Output Characteristics


## Simple dc model

## Typical Output Characteristics



Forward Active region of BJT is analogous to Saturation region of MOSFET Saturation region of BJT is analogous to Triode region of MOSFET

## Better Model of Output Characteristics



With scaled $\mathrm{V}_{\mathrm{CE}}$ axis, transition in saturation very steep

## BJT and MOSFET Comparison

Output Characteristics


- Same general characteristics
- Spacings a bit different (Exponetial vs square law)
- Slope steeper for small $\mathrm{V}_{\mathrm{CE}}$ compared to small $\mathrm{V}_{\mathrm{DS}}$


## Recall MOSFET Operation

Output characteristics


Input characteristics

or equivalently: $\quad I_{G}=0$

## BJT and MOSFET Comparison

Input Characteristics



Did not need to graphically show input characteristics for MOS transistors since $\mathrm{I}_{\mathrm{G}}=0$

## BJT Model

$$
\begin{aligned}
& \mathrm{I}_{\mathrm{B}}=\mathrm{f}_{1}\left(\mathrm{~V}_{\mathrm{BE}}, \mathrm{~V}_{\mathrm{CE}}\right) \\
& \mathrm{I}_{\mathrm{C}}=\mathrm{f}_{2}\left(\mathrm{~V}_{\mathrm{BE}}, \mathrm{~V}_{\mathrm{CE}}\right)
\end{aligned}
$$




Require two graphical representations (or analytical expressions) though vertical axis scales different by factor of $\beta$

Since $I_{B}=f\left(V_{B E}\right)$, can use independent $\left(V_{B E}\right)$ or dependent $\left(I_{B}\right)$ variable for 2-D visualization of 3-dimensional $\mathrm{I}_{\mathrm{C}}$ function

## Improved simple dc model

Typical Output Characteristics


- Projections of these tangential lines all intercept the $-\mathrm{V}_{\mathrm{CE}}$ axis at the same place and this is termed the Early voltage, $\mathrm{V}_{\mathrm{AF}}$ (actually $-\mathrm{V}_{\mathrm{AF}}$ is intercept)
- Typical values of $\mathrm{V}_{\mathrm{AF}}$ are in the 100 V to 200 V range
- Can multiply expression for $I_{C}$ in Forward Active Region by term $\left(1+\frac{V_{C E}}{V_{A F}}\right)$ to account for slope


## Improved simple dc model

(graphically showing only output characteristics)


Need models in saturation and cutoff regions

## Improved simple BJT dc model

Typical Output Characteristics


## Improved simple BJT dc model

Typical Output Characteristics


Need analytical models in saturation and cutoff regions

## Improved simple BJT dc model

Typical Output Characteristics



Recall:
Forward Active region of BJT is analogous to Saturation region of MOSFET Saturation region of BJT is analogous to Triode region of MOSFET

## Improved dc model

(graphically showing only output characteristics)


$$
V_{t}=\frac{k T}{q}
$$

$$
\mathrm{I}_{\mathrm{E}}=-\frac{\mathrm{J}_{\mathrm{S}} A_{E}}{\alpha_{\mathrm{F}}}\left(\mathrm{e}^{\frac{\mathrm{V}_{\mathrm{BE}}}{\mathrm{~V}_{\mathrm{t}}}}-1\right)+\mathrm{J}_{\mathrm{S}} A_{E}\left(\mathrm{e}^{\frac{\mathrm{V}_{\mathrm{BC}}}{\mathrm{~V}_{\mathrm{t}}}}-1\right)
$$

- Valid in All regions of operation
- $V_{\mathrm{AF}}$ effects can be added
- Not mathematically easy to work with

$$
\mathrm{I}_{\mathrm{C}}=\mathrm{J}_{\mathrm{S}} A_{E}\left(\mathrm{e}^{\frac{\mathrm{V}_{\mathrm{BE}}}{\mathrm{~V}_{\mathrm{t}}}}-1\right)-\frac{\mathrm{J}_{\mathrm{S}} A_{E}}{\alpha_{R}}\left(\mathrm{e}^{\frac{\mathrm{V}_{\mathrm{BC}}}{\mathrm{~V}_{\mathrm{t}}}}-1\right)
$$

- Note dependent variables changes $\left\{I_{E}, l_{C}\right\}$
- Termed Ebers-Moll model
- Reduces to previous model in FA region
- Little use in Reverse Active Region


## Improved dc model

(graphically showing only output characteristics)


- Model using $\mathrm{I}_{\mathrm{E}}$ and $\mathrm{I}_{\mathrm{C}}$ as dependent variables
- Valid in All regions of operation

Forward Active

Increasing $\mathrm{V}_{\mathrm{BE}}\left(\right.$ or $\left.\mathrm{I}_{\mathrm{B}}\right)$

$$
\left.\begin{array}{c}
\mathrm{V}_{\mathrm{t}}=\frac{\mathrm{kT}}{\mathrm{q}} \\
\mathrm{I}_{\mathrm{E}}=-\frac{\mathrm{J}_{\mathrm{S}} A_{E}}{\alpha_{\mathrm{F}}}\left(\mathrm{e}^{\frac{\mathrm{V}_{\mathrm{BE}}}{\mathrm{~V}_{\mathrm{t}}}}-1\right)+\mathrm{J}_{\mathrm{S}} A_{E}\left(\mathrm{e}^{\frac{\mathrm{V}_{\mathrm{BC}}}{\mathrm{~V}_{\mathrm{t}}}}-1\right) \\
\mathrm{I}_{\mathrm{C}}=\mathrm{J}_{\mathrm{S}} A_{E}\left(\mathrm{e}^{\frac{\mathrm{V}_{\mathrm{BE}}}{\mathrm{~V}_{\mathrm{t}}}}-1\right)-\frac{\mathrm{J}_{\mathrm{S}} A_{E}}{\alpha_{R}}\left(e^{\frac{\mathrm{V}_{\mathrm{BC}}}{\mathrm{~V}_{\mathrm{t}}}}-1\right)
\end{array}\right\}
$$

- $\mathrm{V}_{\mathrm{AF}}$ effects can be added
- Not mathematically easy to work with
- Note dependent variables changes
- Termed Ebers-Moll model
- Reduces to previous model in FA region
- Little use in Reverse Active Region


## Ebers-Moll model

$$
\begin{aligned}
& \mathrm{I}_{\mathrm{E}}=-\frac{\mathrm{J}_{\mathrm{S}} A_{E}}{\alpha_{\mathrm{F}}}\left(e^{\frac{\mathrm{V}_{\mathrm{BE}}}{\mathrm{~V}_{\mathrm{t}}}}-1\right)+\mathrm{J}_{\mathrm{S}} A_{E}\left(\mathrm{e}^{\frac{\mathrm{V}_{\mathrm{BC}}}{\mathrm{~V}_{\mathrm{t}}}}-1\right) \\
& \mathrm{I}_{\mathrm{C}}=\mathrm{J}_{\mathrm{S}} A_{E}\left(e^{\frac{\mathrm{V}_{\mathrm{BE}}}{\mathrm{~V}_{\mathrm{t}}}}-1\right)-\frac{\mathrm{J}_{\mathrm{S}} A_{E}}{\alpha_{R}}\left(e^{\frac{\mathrm{V}_{\mathrm{BC}}}{\mathrm{~V}_{\mathrm{t}}}}-1\right)
\end{aligned}
$$

Process Parameters: $\left\{J_{S}, \alpha_{F}, \alpha_{R}\right\} \quad V_{t}=\frac{k T}{q}$
Design Parameters: $\left\{\mathrm{A}_{\mathrm{E}}\right\}$
$\alpha_{F}$ is the parameter $\alpha$ discussed earlier $\alpha_{R}$ is termed the "reverse $\alpha$ "

$$
\beta_{F}=\frac{\alpha_{F}}{1-\alpha_{F}} \quad \beta_{R}=\frac{\alpha_{R}}{1-\alpha_{R}} \quad \Longrightarrow \quad \alpha_{F}=\frac{\beta_{F}}{1+\beta_{F}} \quad \alpha_{R}=\frac{\beta_{R}}{1+\beta_{R}}
$$

Typical values for process parameters:

$$
J_{S} \sim 10^{-16} \mathrm{~A} / \mu^{2} \quad \beta_{F} \sim 100, \quad \beta_{R} \sim 0.4
$$

Can substitute for $\alpha_{F}$ and $\alpha_{R}$ in Ebers-Moll model

## Ebers-Moll model

$$
\left.\begin{array}{l}
\mathrm{I}_{\mathrm{E}}=-\frac{\mathrm{J}_{\mathrm{S}} A_{E}}{\alpha_{\mathrm{F}}}\left(\mathrm{e}^{\frac{\mathrm{V}_{\mathrm{BE}}}{\mathrm{v}_{\mathrm{t}}}}-1\right)+\mathrm{J}_{\mathrm{S}} A_{E}\left(\mathrm{e}^{\frac{\mathrm{V}_{\mathrm{BC}}}{\mathrm{~V}_{\mathrm{t}}}}-1\right) \\
\mathrm{I}_{\mathrm{C}}=\mathrm{J}_{\mathrm{S}} A_{E}\left(\mathrm{e}^{\frac{\mathrm{V}_{\mathrm{BE}}}{\mathrm{~V}_{\mathrm{t}}}}-1\right)-\frac{\mathrm{J}_{\mathrm{S}} A_{E}}{\alpha_{R}}\left(e^{\frac{\mathrm{V}_{\mathrm{BC}}}{\mathrm{~V}_{\mathrm{t}}}}-1\right)
\end{array}\right\}
$$

With typical values for process parameters in forward active region $\left(V_{B E} \sim 0.6 V \quad V_{B C} \sim-3 \quad V_{t} \sim 26 \mathrm{mV}\right)$ and if $A_{E}=100 \mu^{2}$

$$
I_{C}=\underbrace{10^{-14}\left(1.05 \times 10^{10}-1\right)-3.6 \times 10^{-14}\left(7.7 \times 10^{-14}-1\right)} \begin{gathered}
\text { Completely dominant }
\end{gathered}
$$

Makes no sense to keep anything other than $I_{C}=J_{S} A_{E} e^{\frac{V_{B E}}{V_{t}}}$ in forward active region

## Ebers-Moll model

$$
\begin{array}{ll}
\text { Ebes-Moll model } & \mathrm{I}_{\mathrm{E}}=-\frac{\mathrm{J}_{\mathrm{S}} A_{E}}{\alpha_{\mathrm{F}}}\left(\mathrm{e}^{\frac{\mathrm{V}_{\mathrm{EE}}}{\mathrm{~V}_{\mathrm{t}}}}-1\right)+\mathrm{J}_{\mathrm{S}} A_{E}\left(\mathrm{e}^{\left.\frac{\mathrm{V}_{\mathrm{BC}}}{\mathrm{~V}_{\mathrm{t}}}-1\right)}\right. \\
\mathrm{V}_{\mathrm{t}}=\frac{\mathrm{kT}}{\mathrm{q}} & \mathrm{I}_{\mathrm{C}}=\mathrm{J}_{\mathrm{S}} A_{E}\left(\mathrm{e}^{\left.\frac{\mathrm{V}_{\mathrm{BE}}}{\mathrm{~V}_{\mathrm{t}}}-1\right)-\frac{\mathrm{J}_{\mathrm{S}} A_{E}}{\alpha_{R}}\left(\mathrm{e}^{\frac{\mathrm{V}_{\mathrm{BC}}}{\mathrm{~V}_{\mathrm{t}}}}-1\right)}\right.
\end{array}
$$

Alternate equivalent expressions for dependent variables $\left\{\left.\right|_{C}, I_{B}\right\}$ defined earlier for Ebers-Moll equations in terms of independent variables $\left\{\mathrm{V}_{\mathrm{BE}}, \mathrm{V}_{\mathrm{CE}}\right\}$ after dropping the " -1 " terms

$$
\begin{gathered}
I_{C}=J_{S} A_{E} e^{\frac{V_{B E}}{V_{t}}}\left(1-\left[\frac{1+\beta_{R}}{\beta_{R}}\right] e^{\frac{-V_{C E}}{V_{t}}}\right) \\
I_{B}=J_{S} A_{E} e^{\frac{V_{B E}}{V_{t}}}\left(\frac{1}{\beta_{F}}-\frac{1}{\beta_{R}} e^{\frac{-V_{C E}}{V_{t}}}\right)
\end{gathered}
$$

No more useful than previous equation but in form consistent with notation Introduced earlier

## Simplified Multi-Region Model

(graphically showing only output characteristics)



- Observe $\mathrm{V}_{\mathrm{CE}}$ around 0.2 V when saturated
- $\mathrm{V}_{\mathrm{BE}}$ around 0.6 V when saturated
- In most applications, exact $\mathrm{V}_{\mathrm{CE}}$ and $\mathrm{V}_{\mathrm{BE}}$ voltage in saturation not critical

Simplified model in saturation:

$$
\left.\begin{array}{l}
V_{\mathrm{BE}}=0.7 \mathrm{~V} \\
\mathrm{~V}_{\mathrm{CE}}=0.2 \mathrm{~V}
\end{array}\right\} \quad \text { Saturation }
$$

## Simplified Multi-Region Model



$$
\begin{aligned}
& I_{C}=J_{S} A_{E} E^{\frac{V_{B E}}{V_{t}}}\left(1+\frac{V_{C E}}{V_{A F}}\right) \\
& I_{B}=\frac{J_{S} A_{E}}{\beta} e^{\frac{V_{B E}}{V_{t}}} \quad V_{t}=\frac{k T}{q}
\end{aligned}
$$

Forward Active
$V_{B E}=0.7 \mathrm{~V}$
Saturation

$$
\mathrm{I}_{\mathrm{C}}=\mathrm{I}_{\mathrm{B}}=0
$$

Cutoff

- This is a piecewise model suitable for analytical calculations
- Can easily extend to reverse active mode but of little use
- Still need conditions for operating in the 3 regions !!


## Simplified Multi-Region Model

"Forward" Regions : $\beta=\beta_{F}$

| $\begin{gathered} I_{C}=J_{S} A_{E} e^{\frac{V_{B E}}{V_{t}}}\left(1+\frac{V_{C E}}{V_{A F}}\right) \\ I_{B}=\frac{J_{S} A_{E}}{\beta} e^{\frac{V_{B E}}{V_{t}}} \end{gathered}$ | Conditions $V_{B E}>0.4 V \quad V_{B C}<0$ | Forward Active |
| :---: | :---: | :---: |
| $\begin{aligned} & V_{\mathrm{BE}}=0.7 \mathrm{~V} \\ & \mathrm{~V}_{\mathrm{CE}}=0.2 \mathrm{~V} \end{aligned}$ | $\mathrm{I}_{\mathrm{C}}<\beta \mathrm{I}_{\mathrm{B}}$ | Saturation |
| $\mathrm{I}_{\mathrm{C}}=\mathrm{I}_{\mathrm{B}}=0$ | $\mathrm{V}_{\mathrm{BE}}<0 \quad \mathrm{~V}_{\mathrm{BC}}<0$ | Cutoff |

Process Parameters: $\left\{J_{S}, \beta, V_{A F}\right\} \quad V_{t}=\frac{k T}{q} \quad$ Design Parameters: $\left\{A_{E}\right\}$

- Process parameters highly process dependent
- $J_{S}$ highly temperature dependent as well, $\beta$ modestly temperature dependent
- This model is dependent only upon emitter area, independent of base and collector area!
- Currents scale linearly with $A_{E}$ and not dependent upon shape of emitter
- A small portion of the operating region is missed with this model but seldom operate in the missing region


## Simplified Multi-Region Model

| Alternate equivalent model |  |  |
| :---: | :---: | :---: |
| $\begin{aligned} & I_{C}=\beta I_{B}\left(1+\frac{V_{C E}}{V_{A F}}\right) \\ & I_{B}=\frac{J_{S} A_{E}}{\beta} e^{\frac{V_{B E}}{V_{t}}} \\ & V_{t}=\frac{k T}{q} \end{aligned}$ | Conditions $\begin{aligned} & V_{\mathrm{BE}}>0.4 \mathrm{~V} \\ & \mathrm{~V}_{\mathrm{BC}}<0 \end{aligned}$ | Forward Active |
| $\begin{aligned} & V_{B E}=0.7 \mathrm{~V} \\ & V_{C E}=0.2 \mathrm{~V} \end{aligned}$ | $\mathrm{I}_{C}<\beta \mathrm{I}_{\mathrm{B}}$ | Saturation |
| $I_{C}=I_{B}=0$ | $\begin{aligned} & V_{\mathrm{BE}}<0 \\ & \mathrm{~V}_{\mathrm{BC}}<0 \end{aligned}$ | Cutoff |

A small portion of the operating region is missed with this model but seldom operate in the missing region

## Further Simplified Multi-Region dc Model

 (by neglecting $\mathrm{V}_{\mathrm{AF}}$ )Forward Active


Adequate when it makes little difference whether $\mathrm{V}_{\mathrm{BE}}=0.6 \mathrm{~V}$ or $\mathrm{V}_{\mathrm{BE}}=0.7 \mathrm{~V}$

## Simplified Multi-Region dc Model

Forward Active


Mathematically

$$
\begin{aligned}
& V_{B E}=0.6 \mathrm{~V} \\
& I_{C}=\beta I_{B}
\end{aligned}
$$

Or, if want to show slope in $\mathrm{I}_{\mathrm{C}}-\mathrm{V}_{\mathrm{CE}}$ characteristics

$$
\begin{aligned}
& \mathrm{V}_{\mathrm{BE}}=0.6 \mathrm{~V} \\
& \mathrm{I}_{\mathrm{C}}=\beta \mathrm{I}_{\mathrm{B}}\left(1+\mathrm{V}_{\mathrm{CE}} / \mathrm{V}_{\mathrm{AF}}\right)
\end{aligned}
$$



$$
\begin{aligned}
& R_{C E}=\frac{V_{A F}}{\beta I_{B Q}} \\
& R_{C E} \text { highly nonlinear }
\end{aligned}
$$

## Further Simplified Multi-Region dc Model

Equivalent Further Simplified Multi-Region Model

$$
\begin{aligned}
& I_{C}=\beta I_{B} \\
& V_{B E}=0.6 \mathrm{~V} \\
& V_{t}=\frac{k T}{q} \\
& V_{B E}=0.7 \mathrm{~V} \\
& V_{C E}=0.2 \mathrm{~V} \\
& I_{C}<\left.\beta\right|_{B} \\
& \mathrm{I}_{\mathrm{C}}=\mathrm{I}_{\mathrm{B}}=0 \\
& \begin{array}{l}
V_{B E}<0 \\
V_{B C}<0
\end{array} \\
& V_{B E}>0.4 \mathrm{~V} \\
& V_{B C}<0 \\
& \text { Saturation } \\
& \text { Cutoff }
\end{aligned}
$$

A small portion of the operating region is missed with this model but seldom operate in the missing region

Conditions for Regions of Operation in Multi-Region Model

$$
\begin{array}{ll}
\mathrm{V}_{\mathrm{BE}}>0.4 \mathrm{~V} & \text { Forward Active } \\
\mathrm{V}_{\mathrm{BC}}<0 & \\
\mathrm{I}_{\mathrm{C}}<\beta \mathrm{I}_{\mathrm{B}} & \text { Saturation } \\
& \\
\mathrm{V}_{\mathrm{BE}}<0 & \text { Cutoff } \\
\mathrm{V}_{\mathrm{BC}}<0 &
\end{array}
$$

Note: One condition is on dependent variables !

Observe that in saturation, $\mathrm{I}_{\mathrm{C}}<\beta \mathrm{I}_{\mathrm{B}}$


Can't condition on independent variables in saturation because they are fixed in the model

Regions of Operation in Independent Parameter Domain used In multi-region models


- Seldom operate in regions excluded in this picture
- Limited use in Reverse Active Mode


## Excessive Power Dissipation if either junction has large forward bias



## Safe regions of operation




Actually cutoff, forward active, and reverse active regions can be extended modestly as shown and multi-region models still are quite good

## Sufficient regions of operation for most applications



## Example: Determine $\mathrm{I}_{\mathrm{C}}$ and $\mathrm{V}_{\text {OUT }}$



Example: Determine $\mathrm{I}_{\mathrm{C}}$ and $\mathrm{V}_{\text {OUT }}$
Solution:

$$
\begin{aligned}
& \mathrm{J}_{\mathrm{s}}=10^{-16} \mathrm{~A} / \mu^{2} \\
& \beta=100
\end{aligned}
$$

1. Guess Forward Active Region (and model)
2. Solve Circuit with Guess

3. Verify model (if necessary)

$$
\begin{aligned}
& I_{B}=\frac{(12-0.6)}{500 \mathrm{~K}} \\
& I_{C}=\beta I_{B}=100 \frac{(12-0.6)}{500 \mathrm{~K}}=2.28 \mathrm{~mA} \\
& V_{\text {OUT }}=12-I_{C} \bullet 4 \mathrm{~K}=2.88 \mathrm{~V}
\end{aligned}
$$

4. Verify FA Region

$$
\begin{aligned}
& V_{B E}=0.6 V>0.4 V \quad V_{\mathrm{BE}}>0.4 \mathrm{~V} \quad \text { and } \quad \mathrm{V}_{\mathrm{BC}}<0 \\
& V_{B C}=0.6 V-2.88 V=-2.28 V<0
\end{aligned}
$$

Verify Passes so solution is valid

$$
\begin{aligned}
& I_{C}=2.28 \mathrm{~mA} \\
& V_{\text {oUT }}=2.88 \mathrm{~V}
\end{aligned}
$$

5. Verify model (if necessary)

Solve again with $\mathrm{V}_{\mathrm{BE}}=0.7 \mathrm{~V}$ Will show $V_{\text {OUT }}=2.96 \mathrm{~V}$ so difference is small
Note solution independent of $J_{S}$ and $A_{E}$

## Example: Determine $\mathrm{I}_{\mathrm{C}}$ and $\mathrm{V}_{\text {out }}$,



Example: Determine $\mathrm{I}_{\mathrm{C}}$ and $\mathrm{V}_{\text {OUT }}$.

## Solution:



1. Guess Forward Active Region
2. Solve Circuit with Guess
3. Verify model (if necessary)


$$
\begin{aligned}
& I_{B}=\frac{(12-0.6)}{50 \mathrm{~K}} \\
& I_{C}=\beta I_{B}=100 \frac{(12-0.6)}{50 \mathrm{~K}}=22.8 \mathrm{~mA} \\
& V_{\text {OUT }}=12-I_{C} \bullet 4 \mathrm{~K}=-79.2 \mathrm{~V}
\end{aligned}
$$

4. Verify FA Region $\mathrm{V}_{\mathrm{BE}}>0.4 \mathrm{~V}$ and $\mathrm{V}_{\mathrm{BC}}<0$

$$
\begin{aligned}
& V_{B E}=0.6 \mathrm{~V}>0.4 \mathrm{~V} \\
& V_{B C}=0.6 \mathrm{~V}--79.2 \mathrm{~V}=+79.8 \mathrm{~V}>0
\end{aligned}
$$

Verify Fails so solution is not valid

Example: Determine $\mathrm{I}_{\mathrm{C}}$ and $\mathrm{V}_{\text {OUT }}$
Solution:

5. Guess Saturation
6. Solve Circuit with Guess
7. Verify model (if necessary)

8. Verify SAT Region

$$
\begin{aligned}
& \beta I_{B}=100 \bullet 228 \mu A=22.8 \mathrm{~mA} \\
& I_{C}=2.95 \mathrm{~mA} \\
& I_{C}=2.95 \mathrm{~mA}<\beta I_{B}=22.8 \mathrm{~mA}
\end{aligned}
$$

Verify SAT Passes so solution is valid

$$
I_{C}=2.95 \mathrm{~mA} \quad V_{\text {OUT }}=0.2 \mathrm{~V}
$$

9. Verify model (if necessary)
(use $\mathrm{V}_{\mathrm{BE}}=0.7 \mathrm{~V}$, no change in output)

## Example: Determine $\mathrm{I}_{\mathrm{C}}$ and $\mathrm{V}_{\text {OUt. }}$. Assume C is large and $\mathrm{V}_{\text {IN }}$ is very small.



Example: Determine $\mathrm{I}_{\mathrm{C}}$ and $\mathrm{V}_{\text {OUt. }}$. Assume C is large and $\mathrm{V}_{\mathrm{IN}}$ is very small.


Solution:

Assume $\mathrm{V}_{\text {IN }}=0$, then no current flows through $C$ so circuit is identical to circuit of previous-previous example so

$$
I_{C}=2.28 \mathrm{~mA} \quad V_{\text {OUT }}=2.88 \mathrm{~V}
$$

Note: If C is large and $\mathrm{V}_{\mathrm{IN}}$ is small sinusoidal signal of sufficiently high frequency, the voltage across $C$ will not change the input so $V_{I N}$ is from an ac viewpoint coupled directly to base. In this case, the circuit will amplify $\mathrm{V}_{\mathbb{I N}}$ and the gain will be very large due to the exponential relationship between $\mathrm{I}_{\mathrm{C}}$ and $\mathrm{V}_{\mathrm{BE}}$.

Example: Determine $\mathrm{I}_{\mathrm{D}}$ and $\mathrm{V}_{\text {OUt. }}$. Assume C is large and $\mathrm{V}_{\text {IN }}$ is very small.


Solution:
Since $\mathrm{I}_{\mathrm{G}}=0$,

$$
V_{G}=\frac{100 K}{600 K} 12 V=2 V
$$

Guess Saturation Region for MOSFET

$$
\begin{aligned}
& \quad V_{G S}>V_{T H} \quad V_{D S}>V_{G S}-V_{T H} \\
& I_{D}=\mu C_{\text {Ox }} \frac{W}{2 L}\left(V_{G S}-V_{T H}\right)^{2} \\
& I_{D}=10^{-4} \frac{45.6}{2}(2-1)^{2}=2.28 \mathrm{~mA} \\
& V_{\text {OUT }}=2.88 \mathrm{~V} \\
& \text { Verify saturation } \quad 2 \mathrm{~V}>1 \mathrm{~V} \quad 2.88 \mathrm{~V}>2 \mathrm{~V}-1 \mathrm{~V}
\end{aligned}
$$

Note: solution dependent upon $\mathrm{W}, \mathrm{L}, \mathrm{V}_{\mathrm{TH}}$, and $\mathrm{uC}_{\mathrm{ox}}$

Example: Determine $\mathrm{I}_{\mathrm{D}}$ and $\mathrm{V}_{\text {OUt. }}$. Assume C is large and $\mathrm{V}_{\text {IN }}$ is very small.


Solution:
Assume $\mathrm{V}_{\mathbb{I N}}=0$, then no current flows through C
$V_{G}=\frac{100 K}{600 K} 12 V=2 V$
Guess Saturation Region for MOSFET

$$
V_{G S}>V_{T H} \quad V_{D S}>V_{G S}-V_{T H}
$$

$I_{D}=\mu C_{O X} \frac{W}{2 L}\left(V_{G S}-V_{T H}\right)^{2}$

$$
I_{D}=10^{-4} \frac{45.6}{2}(2-1)^{2}=2.28 \mathrm{~mA}
$$

$$
V_{\text {OUT }}=2.88 \mathrm{~V}
$$

Verify saturation $2 V>1 V \quad 2.88 V>2 V-1 V$
Note: This circuit has the same current and same $\mathrm{V}_{\text {OUT }}$ as the previous circuit Note: solution dependent upon $\mathrm{W}, \mathrm{L}, \mathrm{V}_{\mathrm{TH}}$, and $u \mathrm{C}_{\mathrm{ox}}$

Example: Determine $\mathrm{I}_{\mathrm{D}}$ and $\mathrm{V}_{\text {OUt. }}$. Assume C is large and $\mathrm{V}_{\mathbb{I N}}$ is very small.


Solution:

Assume $\mathrm{V}_{\text {IN }}=0$, then no current flows through $C$ so circuit is identical to circuit of previous-previous example so

$$
I_{C}=2.28 \mathrm{~mA} \quad V_{\text {OUT }}=2.88 \mathrm{~V}
$$

Note: If C is large and $\mathrm{V}_{\mathbb{I N}}$ is small sinusoidal signal of sufficiently high frequency, the voltage across $C$ will not change so $V_{\text {IN }}$ is from an ac viewpoint coupled directly to gate. In this case, the circuit will amplify $\mathrm{V}_{\mathbb{I N}}$ and the gain will be large due to the square-law relationship between $I_{D}$ and $V_{G S}$.

## Comparison



- Both circuits can serve as amplifiers
- Architectures very similar
- Will be shown later that the bipolar circuit has larger gain because exponential vs square law relationship



## Stay Safe and Stay Healthy !

## End of Lecture 20

